Government of Western Australia
School Curriculum and Standards Authority

MATHEMATICS SPECIALIST

ATAR COURSE

Year 11 syllabus

IMPORTANT INFORMATION

This syllabus is effective from 1 January 2020.

Users of this syllabus are responsible for checking its currency.

Syllabuses are formally reviewed by the School Curriculum and Standards Authority on a cyclical basis, typically every five years.

Content

Overview of mathematics courses

There are six mathematics courses. Each course is organised into four units. Unit 1 and Unit 2 are taken in Year 11 and Unit 3 and Unit 4 in Year 12. The ATAR course examination for each of the three ATAR courses is based on Unit 3 and Unit 4 only.

The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Mathematics Preliminary is a course which focuses on the practical application of knowledge, skills and understandings to a range of environments that will be accessed by students with special education needs. Grades are not assigned for these units. Student achievement is recorded as 'completed' or 'not completed'. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Foundation is a course which focuses on building the capacity, confidence and disposition to use mathematics to meet the numeracy standard for the Western Australian Certificate of Education (WACE). It provides students with the knowledge, skills and understanding to solve problems across a range of contexts, including personal, community and workplace/employment. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Essential is a General course which focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This course provides the opportunity for students to prepare for post-school options of employment and further training.

Mathematics Applications is an ATAR course which focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering questions that involve analysing univariate and bivariate data, including time series data.

Mathematics Methods is an ATAR course which focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

Mathematics Specialist is an ATAR course which provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. The Mathematics Specialist ATAR course contains topics in functions and calculus that build on and deepen the ideas presented in the Mathematics Methods ATAR course, as well as demonstrate their application in many areas. This course also extends understanding and knowledge of statistics and introduces the topics of vectors, complex numbers and matrices. The Mathematics Specialist ATAR course is the only ATAR mathematics course that should not be taken as a stand-alone course.

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics are concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Both mathematics and statistics are widely applicable as models of the world around us and there is ample opportunity for problem-solving throughout the Mathematics Specialist ATAR course. There is also a sound logical basis to this subject, and in mastering the course, students will develop logical reasoning skills to a high level.

The Mathematics Specialist ATAR course provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs and to use mathematical and statistical models more extensively. Topics are developed systematically and lay the foundations for future studies in quantitative subjects in a coherent and structured fashion. Students of the Mathematics Specialist ATAR course will be able to appreciate the true nature of mathematics, its beauty and its functionality.

The Mathematics Specialist ATAR course has been designed to be taken in conjunction with the Mathematical Methods ATAR course. The subject contains topics in functions, calculus, probability and statistics that build on and deepen the ideas presented in the Mathematical Methods ATAR course and demonstrate their application in many areas. Vectors, complex numbers and matrices are introduced. The Mathematics Specialist ATAR course is designed for students with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at university.

For all content areas of the Mathematics Specialist ATAR course, the proficiency strands of the Year 7–10 curriculum continue to be applicable and should be inherent in students' learning of the subject. These strands are Understanding, Fluency, Problem-solving and Reasoning and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency of skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves, freeing up working memory for more complex aspects of problem-solving. In the Mathematics Specialist ATAR course, the formal explanation of reasoning through mathematical proof takes on an important role and the ability to present the solution of any problem in a logical and clear manner is of paramount importance. The ability to transfer skills learned to solve one class of problem, for example, integration, to solve another class of problem, such as in biology, kinematics or statistics, is a vital part of mathematics learning in this subject.

The Mathematics Specialist ATAR course is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit blends algebraic and geometric thinking. In this subject, there is a progression of content, applications, level of sophistication and abstraction. For example, in Unit 1, vectors for two-dimensional space are introduced and in Unit 3, vectors are studied for three-dimensional space. The Unit 3 vector topic leads to the establishment of the equations of lines and planes, and this in turn prepares students for an introduction to solving simultaneous equations in three variables. The study of calculus, which is developed in the Mathematical Methods ATAR course, is applied in vectors in Unit 3 and applications of calculus and statistics in Unit 4.

Aims

The Mathematics Specialist ATAR course aims to develop students':

- understanding of concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- ability to solve applied problems using concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- capacity to choose and use technology appropriately
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- ability to construct proofs.

Organisation

This course is organised into a Year 11 syllabus and a Year 12 syllabus. The cognitive complexity of the syllabus content increases from Year 11 to Year 12.

Structure of the syllabus

The Year 11 syllabus is divided into two units, each of one semester duration, which are typically delivered as a pair. The notional time for each unit is 55 class contact hours.

Organisation of content

Unit 1

Contains the three topics:

1.1 Combinatorics

1.2 Vectors in the plane

1.3 Geometry

The three topics in Unit 1 complement the content of the Mathematics Methods ATAR course. The proficiency strand of Reasoning, from the Year 7–10 curriculum, is continued explicitly in the topic Geometry through a discussion of developing mathematical arguments. This topic also provides the opportunity to summarise and extend students' studies in Euclidean Geometry, knowledge which is of great benefit in the later study of topics such as vectors and complex numbers. The topic Combinatorics provides techniques that are very useful in many areas of mathematics, including probability and algebra. The topic Vectors in the plane provides new perspectives on working with two-dimensional space and serves as an introduction to techniques which can be extended to three-dimensional space in Unit 3. These three topics considerably broaden students' mathematical experience and therefore begin an awakening to the breadth and utility of the subject. They also enable students to increase their mathematical flexibility and versatility.

Unit 2

Contains the three topics:

2.1 Trigonometry

2.2 Matrices

2.3 Real and complex numbers

In Unit 2, Matrices provide new perspectives for working with two-dimensional space and Real and complex numbers provides a continuation of the study of numbers. The topic Trigonometry contains techniques that are used in other topics in both this unit and Units 3 and 4. All topics develop students' ability to construct mathematical arguments. The technique of proof by the principle of mathematical induction is introduced in this unit.

Each unit includes:

- a unit description – a short description of the focus of the unit
- learning outcomes – a set of statements describing the learning expected as a result of studying the unit
- unit content – the content to be taught and learned.

Role of technology

It is assumed that students studying the Mathematics Specialist ATAR course will have access to an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in this course.

Progression from the Year 7–10 curriculum

For all content areas of this syllabus, the proficiency strands of the Year 7–10 curriculum continue to apply and should be inherent in students' learning of the course. The strands of Understanding, Fluency, and Problem solving and Reasoning are essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving. In this syllabus, the formal explanation of reasoning through mathematical proof takes an important role. The ability to present the solution of any problem in a logical and clear manner is of paramount significance. The ability to transfer skills learned to solve one class of problem, such as trigonometry, to solve another class of problems, such as those in vectors or complex numbers, is a vital part of mathematics learning in this course. In order to study this syllabus, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended as preparation for the Mathematics Specialist ATAR course.

- ACMMG272: Prove and apply angle and chord properties of circles
- ACMMG273: Establish the sine, cosine and area rules for any triangle, and solve related problems.

Representation of the general capabilities

The general capabilities encompass the knowledge, skills, behaviours and dispositions that will assist students to live and work successfully in the twenty-first century. Teachers may find opportunities to incorporate the capabilities into the teaching and learning program for Mathematics Specialist. The general capabilities are not assessed unless they are identified within the specified unit content.

Literacy

Literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy

Students who undertake this course will continue to develop their numeracy skills at a more sophisticated level, making decisions about the relevant mathematics to use, following through with calculations selecting appropriate methods and being confident of their results. This course contains topics that will equip students for the ever-increasing demands of the information age, developing the skills of critical evaluation of numerical information. Students will enhance their numerical operation skills by application in counting techniques problems, as well as in other topics such as the algebra of complex numbers, vectors, and with matrix arithmetic.

Information and communication technology capability

Students use information and communication technology (ICT) both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved, such as for statistical analysis, generation of algorithms, manipulation and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability

Students develop personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision making.

The elements of personal and social competence relevant to mathematics mainly include the application of mathematical skills for their decision making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical understanding

Students develop Ethical understanding in mathematics through decision making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results, and the social responsibility associated with teamwork and attribution of input.

The areas relevant to mathematics include issues associated with ethical decision making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and ethical understanding. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding

Students understand mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Representation of the cross-curriculum priorities

The cross-curriculum priorities address contemporary issues which students face in a globalised world. Teachers will find opportunities to incorporate the priorities into the teaching and learning program for the Mathematics Specialist ATAR course. The cross-curriculum priorities are not assessed unless they are identified within the specified unit content.

Aboriginal and Torres Strait Islander histories and cultures

Mathematics courses value the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples' past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples' histories and cultures.

Asia and Australia's engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Sustainability

Each of the mathematics courses provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 1

Unit description

Unit 1 of the Mathematics Specialist ATAR course contains three topics: Combinatorics, Vectors in the plane, and Geometry that complement the content of the Mathematical Methods ATAR course. The proficiency strand, Reasoning, of the Year 7–10 curriculum is continued explicitly in Geometry through a discussion of developing mathematical arguments. While these ideas are illustrated through deductive Euclidean geometry in this topic, they recur throughout all topics in the Mathematics Specialist ATAR course. Geometry also provides the opportunity to summarise and extend students' studies in Euclidean Geometry. An understanding of this topic is of great benefit in the study of later topics in the course, including vectors and complex numbers.

Vectors in the plane provides new perspectives for working with two-dimensional space and serves as an introduction to techniques that will be extended to three-dimensional space in Unit 3.

Combinatorics provides techniques that are useful in many areas of mathematics, including probability and algebra. All topics develop students' ability to construct mathematical arguments.

The three topics considerably broaden students' mathematical experience and therefore begin an awakening to the breadth and utility of the course. They also enable students to increase their mathematical flexibility and versatility.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in combinatorics, geometry and vectors
- apply reasoning skills and solve problems in combinatorics, geometry and vectors
- communicate their arguments and strategies when solving problems
- construct proofs in a variety of contexts, including algebraic and geometric
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Unit content

This unit includes the knowledge, understandings and skills described below.

Topic 1.1: Combinatorics (11 hours)

Permutations (ordered arrangements)

1.1.1 solve problems involving permutations
1.1.2 use the multiplication and addition principle
1.1.3 use factorial notation and ${}^nP_r$
1.1.4 solve problems involving permutations involving restrictions with or without repeated objects

The inclusion-exclusion principle for the union of two sets and three sets

1.1.5 determine and use the formulas for finding the number of elements in the union of two and the union of three sets

The pigeon-hole principle

1.1.6 solve problems and prove results using the pigeon-hole principle

Combinations (unordered selections)

1.1.7 solve problems involving combinations

1.1.8 use the notation $\binom{n}{r}$ or ${}^{n}C_{r}$

1.1.9 derive and use associated simple identities associated with Pascal's triangle

Topic 1.2: Vectors in the plane (22 hours)

Representing vectors in the plane by directed line segments

1.2.1 examine examples of vectors, including displacement and velocity

1.2.2 define and use the magnitude and direction of a vector

1.2.3 represent a scalar multiple of a vector

1.2.4 use the triangle and parallelogram rules to find the sum and difference of two vectors

Algebra of vectors in the plane

1.2.5 use ordered pair notation and column vector notation to represent a vector

1.2.6 define unit vectors and the perpendicular unit vectors $\mathbf{i}$ and $\mathbf{j}$

1.2.7 express a vector in component form using the unit vectors $\mathbf{i}$ and $\mathbf{j}$

1.2.8 examine and use addition and subtraction of vectors in component form

1.2.9 define and use multiplication of a vector by a scalar in component form

1.2.10 define and use scalar (dot) product

1.2.11 apply the scalar product to vectors expressed in component form

1.2.12 examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular

1.2.13 define and use projection of vectors

1.2.14 solve problems involving displacement, force and velocity involving the above concepts

Topic 1.3: Geometry (22 hours)

The nature of proof

1.3.1 use implication, converse, equivalence, negation, inverse, contrapositive

1.3.2 use proof by contradiction

1.3.3 use the symbols for implication (⇒), equivalence (⇔)

1.3.4 use the quantifiers 'for all' ∀ and 'there exists' ∃.

1.3.5 use examples and counter-examples

Circle properties, including proof and use

1.3.6 an angle in a semicircle is a right angle

1.3.7 the size of the angle at the centre subtended by an arc of a circle is twice the size of the angle at the circumference subtended by the same arc

1.3.8 angles at the circumference of a circle subtended by the same arc are equal

1.3.9 the opposite angles of a cyclic quadrilateral are supplementary

1.3.10 chords of equal length subtend equal angles at the centre, and conversely, chords subtending equal angles at the centre of a circle have the same length

1.3.11 the angle in the alternate segment theorem

1.3.12 when two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord

1.3.13 when a secant (meeting the circle at A and B) and a tangent (meeting the circle at T) are drawn to a circle from an external point M, the square of length of the tangent equals the product of the lengths to the circle on the secant ($AM \times BM = TM^2$)

1.3.14 suitable converses of some of the above results

1.3.15 solve problems determining unknown angles and lengths and prove further results using the results listed above

Geometric vectors in the plane, including proof and use

1.3.16 the diagonals of a parallelogram intersect at right angles if, and only if, it is a rhombus

1.3.17 the midpoints of the sides of a quadrilateral join to form a parallelogram

1.3.18 the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides

Unit 2

Unit description

Unit 2 of the Mathematics Specialist ATAR course contains three topics: Trigonometry, Matrices, and Real and complex numbers.

Trigonometry contains techniques that are used in other topics in both this unit and Unit 3. Real and complex numbers provides a continuation of students' study of numbers, and the study of complex numbers is continued in Unit 3. This topic also contains a section on proof by mathematical induction. The study of Matrices is undertaken, including applications to linear transformations of the plane.

Access to technology to support the computational aspects of these topics is assumed.

Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in trigonometry, real and complex numbers, and matrices
- apply reasoning skills and solve problems in trigonometry, real and complex numbers, and matrices
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Unit content

This unit includes the knowledge, understandings and skills described below.

Topic 2.1: Trigonometry (16 hours)

The basic trigonometric functions

2.1.1 determine all solutions of $f(a(x-b))=c$ where f is one of sine, cosine or tangent

2.1.2 graph functions with rules of the form $y=f(a(x-b))+c$ where f is one of sine, cosine, or tangent

Compound angles

2.1.3 prove and apply the angle sum, difference, and double angle identities

The reciprocal trigonometric functions, secant, cosecant and cotangent

2.1.4 define the reciprocal trigonometric functions; sketch their graphs and graph simple transformations of them

Trigonometric identities

2.1.5 prove and apply the Pythagorean identities

2.1.6 prove and apply the identities for products of sines and cosines expressed as sums and differences

2.1.7 convert sums $a \cos x + b \sin x$ to $R \cos(x \pm \alpha)$ or $R \sin(x \pm \alpha)$ and apply these to sketch graphs; solve equations of the form $a \cos x + b \sin x = c$

2.1.8 prove and apply other trigonometric identities such as $\cos 3x = 4 \cos^3 x - 3 \cos x$

Applications of trigonometric functions to model periodic phenomena

2.1.9 model periodic motion using sine and cosine functions and understand the relevance of the period and amplitude of these functions in the model

Topic 2.2: Matrices (19 hours)

Matrix arithmetic

2.2.1 apply matrix definition and notation

2.2.2 define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity, and inverse

2.2.3 calculate the determinant and inverse of 2×2 matrices and solve matrix equations of the form $AX = B$, where A is a 2×2 matrix and X and B are column vectors

Transformations in the plane

2.2.4 examine translations and their representation as column vectors

2.2.5 define and use basic linear transformations: dilations of the form $(x,y) \longrightarrow (\lambda_1 x, \lambda_2 y)$, rotations about the origin and reflection in a line that passes through the origin and the representations of these transformations by 2×2 matrices

2.2.6 apply these transformations to points in the plane and geometric objects

2.2.7 define and use composition of linear transformations and the corresponding matrix products

2.2.8 define and use inverses of linear transformations and the relationship with the matrix inverse

2.2.9 examine the relationship between the determinant and the effect of a linear transformation on area

2.2.10 establish geometric results by matrix multiplications; for example: show that the combined effect of 2 reflections is a rotation

Systems of linear equations

2.2.11 interpret the matrix form of a system of linear equations in two variables and use matrix algebra to solve a system of linear equations

Topic 2.3: Real and complex numbers (20 hours)

Proofs involving numbers

2.3.1 prove simple results involving numbers

Rational and irrational numbers

2.3.2 express rational numbers as terminating or eventually recurring decimals and vice versa

2.3.3 prove irrationality by contradiction for numbers such as $\sqrt{2}$

An introduction to proof by mathematical induction

2.3.4 develop the nature of inductive proof, including the ‘initial statement’ and inductive step

2.3.5 prove results for sums, such as $1+4+9...+n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer *n*

2.3.6 prove divisibility results, such as $3^{2n+4}-3^{2n}$ is divisible by 5 for any positive integer *n*

Complex numbers

2.3.7 define the imaginary number *i* as a root of the equation $x^2=-1$

2.3.8 represent complex numbers in the rectangular form; *a* + *bi* where *a* and *b* are the real and imaginary parts

2.3.9 determine and use complex conjugates

2.3.10 perform complex number arithmetic: addition, subtraction, multiplication and division

The complex plane

2.3.11 consider complex numbers as points in a plane, with real and imaginary parts, as Cartesian coordinates

2.3.12 examine addition of complex numbers as vector addition in the complex plane

2.3.13 develop and use the concept of complex conjugates and their location in the complex plane

Roots of equations

2.3.14 use the general solution of real quadratic equations

2.3.15 determine complex conjugate solutions of real quadratic equations

2.3.16 determine linear factors of real quadratic polynomials

School-based assessment

The Western Australian Certificate of Education (WACE) Manual contains essential information on principles, policies and procedures for school-based assessment that needs to be read in conjunction with this syllabus.

Teachers design school-based assessment tasks to meet the needs of students. The table below provides details of the assessment types for the Mathematics Specialist ATAR Year 11 syllabus and the weighting for each assessment type.

Assessment table – Year 11

Type of assessment	Weighting
Response Students respond using knowledge of mathematical facts, concepts and terminology, applying problem-solving skills and algorithms. Response tasks can include: tests, assignments, quizzes and observation checklists. Tests are administered under controlled and timed conditions.	40%
Investigation Students use the mathematical thinking process to plan, research, conduct and communicate the findings of an investigation. They can investigate problems to identify the underlying mathematics, or select, adapt and apply models and procedures to solve problems. This assessment type provides for the assessment of use the mathematical thinking process using course-related knowledge and skills and modelling skills. Evidence can include: observation and interview, written work or multimedia presentations.	20%
Examination Students apply mathematical understanding and skills to analyse, interpret and respond to questions and situations. Examinations provide for the assessment of conceptual understandings, knowledge of mathematical facts and terminology, problem-solving skills, and the use of algorithms. Examination questions can range from those of a routine nature, assessing lower level concepts, through to those that require responses at the highest level of conceptual thinking. Students can be asked questions for which they may need to construct proofs and make conjectures. Typically conducted at the end of each semester and/or unit. In preparation for Unit 3 and Unit 4, the examination should reflect the examination design brief included in the ATAR Year 12 syllabus for this course. Where a combined assessment outline is implemented, the Semester 2 examination should assess content from both Unit 1 and Unit 2. However, the combined weighting of Semester 1 and Semester 2 should reflect the respective weightings of the course content as a whole.	40%

Teachers are required to use the assessment table to develop an assessment outline for the pair of units (or for a single unit where only one is being studied).

The assessment outline must:

- include a set of assessment tasks
- include a general description of each task
- indicate the unit content to be assessed
- indicate a weighting for each task and each assessment type
- include the approximate timing of each task (for example, the week the task is conducted, or the issue and submission dates for an extended task).

In the assessment outline for the pair of units:

- each assessment type must be included at least twice
- the response type must include a minimum of two tests.

In the assessment outline where a single unit is being studied:

- each assessment type must be included at least once
- the response type must include at least one test.

The set of assessment tasks must provide a representative sampling of the content for Unit 1 and Unit 2.

Assessment tasks not administered under test/controlled conditions require appropriate validation/authentication processes. This may include observation, annotated notes, checklists, interview, presentations or in-class tasks assessing related content and processes.

Grading

Schools report student achievement in terms of the following grades:

Grade	Interpretation
A	Excellent achievement
B	High achievement
C	Satisfactory achievement
D	Limited achievement
E	Very low achievement

The teacher prepares a ranked list and assigns the student a grade for the pair of units (or for a unit where only one unit is being studied). The grade is based on the student's overall performance as judged by reference to a set of pre-determined standards. These standards are defined by grade descriptions and annotated work samples. The grade descriptions for the Mathematics Specialist ATAR Year 11 syllabus are provided in Appendix 1. They can also be accessed, together with annotated work samples, through the Guide to Grades link on the course page of the Authority website at www.scsa.wa.edu.au

To be assigned a grade, a student must have had the opportunity to complete the education program, including the assessment program (unless the school accepts that there are exceptional and justifiable circumstances).

Refer to the WACE Manual for further information about the use of a ranked list in the process of assigning grades.

Appendix 1 – Grade descriptions Year 11

A	**Identifies and organises relevant information** Identifies and organises relevant information from complex sources, for example descriptive passages, labelled diagrams or tables of data. Recognises various vector and trigonometric functions and their domain and range. Identifies key elements in ambiguous data, such as how the domain of an angle affects the sign of the trigonometric ratios. Identifies key information from scattered sources such as interpreting the physical contexts involving combinations or permutations and solving related problems.
	Chooses effective models and methods and carries the methods through correctly Chooses and uses the correct technique or model in unpractised situations. Carries deductive reasoning and extended responses through clearly. Simplifies complicated fractions and works efficiently with algebraic expressions in fraction form. Translates fluently between representations, such as geometric vector diagrams to algebraic expressions. Uses a calculator appropriately for calculation, algebra and graphing, and highlights less obvious features of graphs, such as asymptotes or end points.
	Follows mathematical conventions and attends to accuracy Uses correct notation with vectors, matrices, combinatorics and complex numbers. Defines variables and parameters to suit the context. Draws clear geometric and vector diagrams with appropriate scales and labels. Works well with exact values such as surds, radian values or factorial notation, and recognises the difference between open and closed intervals. Uses appropriate logical operators and geometric notation when setting out geometric proofs.
	Links mathematical results to data and contexts to reach reasonable conclusions Pays attention to units in all tasks, giving answers to the correct degree of accuracy, and uses radian measure when appropriate. Takes account of the domain with time as the independent variable defined in vector problems, or by the context of the question, and excludes any results outside the domain.
	Communicates mathematical reasoning, results and conclusions Shows the main steps in mathematical reasoning in a logical sequence. Sets out geometric proofs in a logical and clear manner. Draws diagrams and defines appropriate vertices and angles to use in the working of a problem. Relates the result of a problem to the context of the question by using the correct units and any related notation, such as vector notation.

B	**Identifies and organises relevant information** Identifies and organises relevant information from concentrated or scattered sources. Draws and labels diagrams from written instructions. Identifies key elements in ambiguous data, for example distinguishing the time required as the time of day rather than the time elapsed.
	Chooses effective models and methods and carries the methods through correctly Carries deductive reasoning and extended responses through and applies various rules, for example the Distributive Law or Commutative Law to simplify vector dot products. Generalises mathematical structures when determining the transformational effects of the parameters a, b, c and d, in trigonometric equations, for example, $y=a\sin b(x+c)+d$. Moves between representations in unpractised ways, such as drawing vectors in the Cartesian plane to determine the resultant. Converts multi-dimensional units such as kilometres per hour to meters per second. Uses a calculator appropriately for vectors, geometry and graphing and pays attention to features of graphs, such as amplitude and phase shift. Carries through accurately with vector algebra.
	Follows mathematical conventions and attends to accuracy. Rounds to suit contexts, specified accuracies and boundary values on occasions, for example rounds 3.70 hours to 3 hours 42 minutes (i.e. to the nearest minute). Takes note of the laws used when dealing with matrix algebra. Interprets the information on diagrams by using the various geometric symbols such as parallel lines or congruent sides.
	Links mathematical results to data and contexts to reach reasonable conclusions Attends to units in extended tasks, such as determining the distance travelled in a set time given the vector equation of motion. Can switch logical statements, for example to their converse or contrapositive.
	Communicates mathematical reasoning, results and conclusions Shows main steps in reasoning when setting out a geometric proof. Identifies the period and scale factor of a tangent graph $y=a\tan bx$ from a given graph.

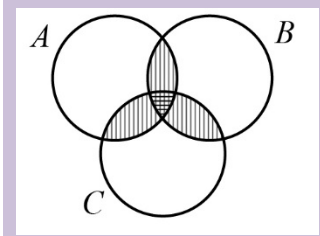
C	**Identifies and organises relevant information** Identifies and organises relevant information that is relatively narrow in scope, for example, uses the information in diagrams supplied with the problem. Identifies the correct trigonometric formulas in straightforward situations. Writes the component form of vectors from a Cartesian diagram.
	Chooses effective models and methods and carries the methods through correctly Answers structured questions that require short responses, such as solving simple vector diagrams or composing functions. Makes common sense connections in practical diagrams involving navigation or simple bearings. Translates between representations in practised ways, for example matches trigonometric graphs to their stated equation, draws vector diagrams on a Cartesian plane from a given simple component form. Uses a calculator appropriately for calculation, combinatorics or straightforward graphing. Shows basic features on sketches of graphs located using a calculator.
	Follows mathematical conventions and attends to accuracy Defines introduced variables, for example, labels a diagram and allocates a variable to the length of an unknown side or the size of angle. Applies conventions for diagrams and graphs by labelling points with upper case letters, for example A, B and using arrows to convey the direction of a vector. Rounds to suit contexts and specified accuracies in short responses, for example, rounding angles to the nearest degree in navigation problems.
	Links mathematical results to data and contexts to reach reasonable conclusions Recognises specified conditions in short responses and includes the SI units with an answer when required, for example, $\text{velocity (ms}^{-1})$ x $\text{time (s)} = \text{distance (m)}$. Attends to radian measure or degrees as is appropriate in trigonometry problems.
	Communicates mathematical reasoning, results and conclusions Shows working and sets out algebraic solutions correctly in short response questions. Makes clear sketches of simple functions, including some detail. Draws simple diagrams to help with solving problems of space and measurement.

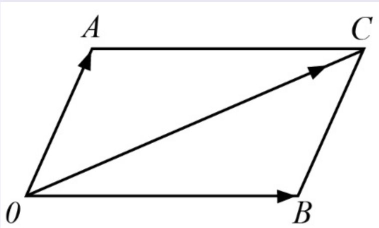
D	**Identifies and organises relevant information** Identifies and organises relevant information that is narrow in scope. Converts degrees to radians and vice versa.
	Chooses effective models and methods and carries the methods through correctly Answers familiar, structured questions that require short responses, for example locating the intersection of two graphs. Reads the components of vectors from a diagram. Applies mathematics in practised ways to calculate magnitude of vectors. Simplifies fractions such as $\frac{5\pi}{6} \times \frac{180^0}{\pi}$.
	Follows mathematical conventions and attends to accuracy Applies conventions for graphs to label axes, set up a scale, and label different graphs on the same axes with some errors. Rounds to suit contexts and specified accuracies in short responses, such as rounding to a stated degree of accuracy, for example x = 30.86 km (two decimal places).
	Links mathematical results to data and contexts to reach reasonable conclusions Does not recognise specified conditions to identify the need to give exact value answers with conversions of radian measure to degrees. Attends to units in short responses when prompted, for example, the unit measure when calculating the magnitude of a vector.
	Communicates mathematical reasoning, results and conclusions Uses correct vector notation most of the time. Expresses the component parts **i** and **j** of a vector during calculations and in manipulations of terms or expressions. Obeys conventions for vector diagrams. Labels simple geometry diagrams and expands combination and permutation expressions with some errors.

E	Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade.

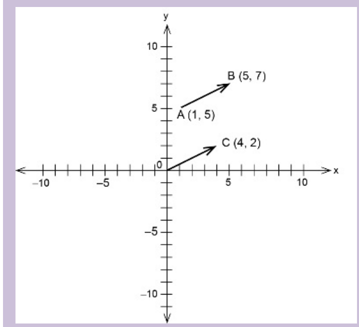
Appendix 2 – Glossary

This glossary is provided to enable a common understanding of the key terms in this syllabus.

UNIT 1	
Combinatorics	
Arranging n objects in an ordered list	The number of ways to arrange n different objects in an ordered list is given by $n(n-1)(n-2)\times\ldots\times3\times2\times1=n!$
Combinations (Selections)	The number of selections of n objects taken r at a time (that is, the number of ways of selecting r objects out of n) is denoted by ${}^nC_r=\binom{n}{r}=\frac{n!}{r!(n-r)!}$
Inclusion – exclusion principle	Suppose A and B are subsets of a finite set X then $\lvert A\cup B\rvert = \lvert A\rvert + \lvert B\rvert - \lvert A\cup B\rvert$ Suppose A, B and C are subsets of a finite set X then  $\lvert A\cup B\cup C\rvert = \lvert A\rvert + \lvert B\rvert + \lvert C\rvert - \lvert A\cap B\rvert - \lvert A\cap C\rvert - \lvert B\cap C\rvert + \lvert A\cap B\cap C\rvert$ This result can be generalised to 4 or more sets.
Multiplication principle	Suppose a choice is to be made in two stages. If there are a choices for the first stage and b choices for the second stage, no matter what choice has been made at the first stage, then there are $a\times b$ choices altogether. If the choice is to be made in n stages and if for each i, there are a_i choices for the i^{th} stage then there are $a_1\times a_2\times a_3\ldots a_n$ choices altogether.
Pascal's triangle	Pascal's triangle is an arrangement of numbers. In general the n^{th} row consists of the binomial coefficients ${}^nC_r=\binom{n}{r}$ where n is the row number $n=0,1,2\ldots$ and r is the term number 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1 1 6 15 20 15 6 1 1 7 21 35 35 21 7 1 In Pascal's triangle any term is the sum of the two terms 'above' it.

Pascal's triangle	For example 10 = 4 + 6. Identities include: The recurrence relation, ${}^{n}C_{k} = {}^{n-1}C_{k-1} + {}^{n-1}C_{k}$ $= \frac{n}{k}\,{}^{n-1}C_{k-1}$
Permutations	A permutation of n objects is an arrangement or rearrangement of n objects where order is important. The number of arrangements of n different objects is $n!$ The number of permutations of n objects taken r at a time is denoted by ${}^{n}P_{r}$ and ${}^{n}P_{r} = n(n-1)(n-2)...(n-r+1) = \frac{n!}{(n-r)!}$ The number of permutations of n objects of which r_1 are identical, another r_2 are identical, r_3 are identical, and so on... $= \frac{n!}{r_1!r_2!r_3!\ ...}$
Pigeon-hole principle	If there are n pigeon holes and $n+1$ pigeons to go into them, then at least one pigeon hole must get 2 or more pigeons.
Vectors in the plane	
Addition of vectors (see Vector for definition and notation)	Given vectors **a** and **b** let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be directed line segments that represent **a** and **b**. They have the same initial point O. The sum of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is the directed line segment $\overrightarrow{OC}$ where C is a point such that $OACB$ is a parallelogram. This is known as the **parallelogram rule**.  If $\mathbf{a} = (a_1, a_2)$ and $\mathbf{b} = (b_1, b_2)$ then $\mathbf{a}+\mathbf{b} = (a_1+b_1, a_2+b_2)$ In component form if $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j}$ then $\mathbf{a}+\mathbf{b} = (a_1+b_1)\mathbf{i} + (a_2+b_2)\mathbf{j}$
Magnitude of a vector (see Vector for definition and notation)	The magnitude of a vector **a** is the length of any directed line segment that represents **a.** It is denoted by $\lvert\mathbf{a}\rvert$.
Multiplication by a scalar	Let **a** be a non-zero vector and k a positive real number (scalar) then the scalar multiple of **a** by k is the vector $k\mathbf{a}$ which has magnitude $\lvert k\rvert\lvert\mathbf{a}\rvert$ and the same direction as **a.** If k is a negative real number, then $k\mathbf{a}$ has magnitude $\lvert k\rvert\,\lvert\mathbf{a}\rvert$ and but is directed in the opposite direction to **a**. (see negative of a vector) Some properties of scalar multiplication are: $k(\mathbf{a}+\mathbf{b}) = k\mathbf{a}+k\mathbf{b}$ $h(k(\mathbf{a})) = (hk)\mathbf{a}$ $1\mathbf{a} = \mathbf{a}$

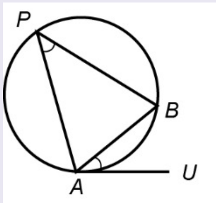
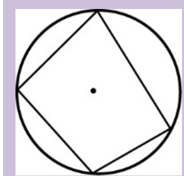
Negative of a vector (see Vector for definition and notation)	Given $\overrightarrow{OA}$ is a directed line segment representing vector **a.** The negative of **a**, denoted by –**a,** is the vector represented by $\overrightarrow{AO}$. The following are properties of vectors involving negatives: **a** + (–**a) =** (–**a**) + **a** = **0** –(–**a**) = **a**
Properties of vector addition:	$\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$ (commutative law) $(\mathbf{a}+\mathbf{b})+\mathbf{c}=\mathbf{a}+(\mathbf{b}+\mathbf{c})$ (associative law) $\mathbf{a}+\mathbf{0}=\mathbf{0}+\mathbf{a}=\mathbf{a}$ (Zero vector) $\mathbf{a}+(-\mathbf{a})=\mathbf{0}$ (Opposite vectors)
Scalar product (see Vector for definition and notation)	**a** = (a_1, a_2) and **b =** (b_1, b_2) then the scalar product $\mathbf{a}\cdot\mathbf{b}$ is the real number $a_1 b_1 + a_2 b_2$. The geometrical interpretation of this number is $\mathbf{a}\cdot\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ where θ is the angle 'between' **a** and **b** When expressed in **i**, **j**, notation, if **a** = $a_1\mathbf{i} + a_2\mathbf{j}$ and **b** = $b_1\mathbf{i} + b_2\mathbf{j}$ then $\mathbf{a}\cdot\mathbf{b} = a_1 b_1 + a_2 b_2$ *Note* $\lvert\mathbf{a}\rvert = \sqrt{\mathbf{a}\cdot\mathbf{a}}$
Subtraction of vectors (see Vector for definition and notation)	To subtract vector **b** from vector **a** we add the negative of **b** to **a** : $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$
Unit vector (see Vector for definition and notation)	A unit vector is a vector with magnitude 1. Given any vector **a** the unit vector in the same direction as **a** is $\dfrac{\mathbf{a}}{\lvert\mathbf{a}\rvert}$. This vector is often denoted as $\hat{\mathbf{a}}$.
Vector projection (see Vector for definition and notation)	Let **a** and **b** be two vectors and write θ for the angle between them. The projection of vector **a** on vector **b** is the vector $\lvert\mathbf{a}\rvert \cos\theta\, \hat{\mathbf{b}} = (\mathbf{a}\cdot\hat{\mathbf{b}})\hat{\mathbf{b}}$ where $\hat{\mathbf{b}}$ is the unit vector in the direction of **b**. This projection is also given by the formula = $\dfrac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\mathbf{b}$.
Vector	In Physics, the name vector is used to describe a physical quantity like velocity or force that has a magnitude and direction. A vector is an entity **a** which has a given length (magnitude) and a given direction. If $\overrightarrow{AB}$ is a directed line segment with this length and direction, then we say that $\overrightarrow{AB}$ represents **a.** If $\overrightarrow{AB}$ and $\overrightarrow{CD}$ represent the same vector, they are parallel and have the same length. The **zero vector** is the vector with length zero. In two dimensions, every vector can be represented by a directed line segment which begins at the origin.

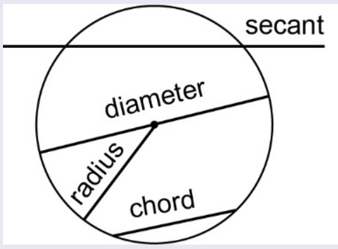
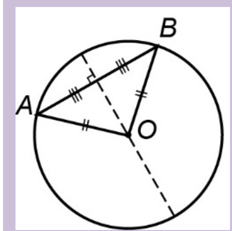
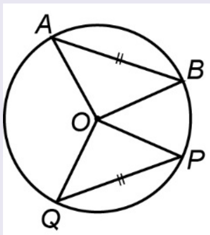
Vector	For example, the vector from $A\ (1,5)\ to\ B\ (5,7)$ can be represented by the directed line segment $\overrightarrow{OC}$ where C is the point $(4,2)$ The ordered pair notation for a vector uses the co-ordinates of the end point of this directed line segment beginning at the origin to denote the vector, so $\overrightarrow{AB}$= $(4,2)$ in ordered pair notation. The same vector can be represented in column vector notation as $\begin{pmatrix}4\\2\end{pmatrix}$. 

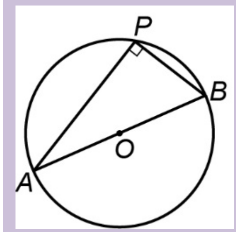
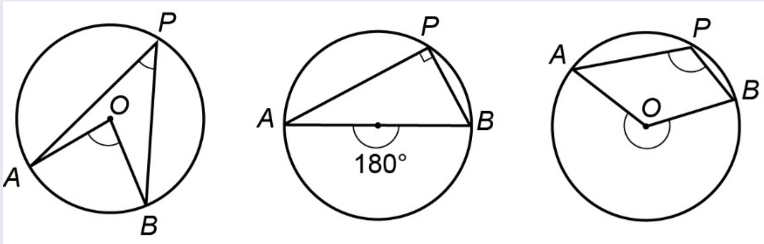
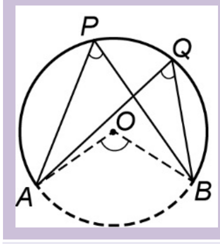
Glossary for Proof

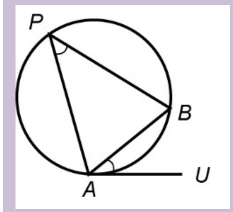
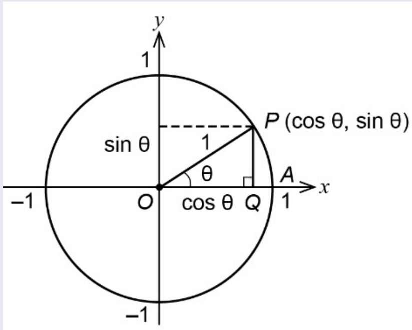
Contradiction – Proof by	Assume the opposite (**negation**) of what you are trying to prove. Then proceed through a logical chain of argument until you reach a demonstrably false conclusion. Since all the reasoning is correct and a false conclusion has been reached the only thing that could be wrong is the initial assumption. Therefore the original statement is true. For example: the result $\sqrt{2}$ is irrational can be proved in this way by first assuming $\sqrt{2}$ is rational. The following are examples of results that are often proved by contradiction: If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic. If an interval (line segment) subtends equal angles at two points on the same side of the interval (line segment), then the two points and the endpoints of the interval are concyclic.
Converse of a statement	The converse of the statement 'If P then Q' is 'If Q then P' Symbolically the converse of P $\Rightarrow$ Q is: Q $\Rightarrow$ P or P $\Leftarrow$ Q The converse of a true statement need not be true. Examples: **Statement:** If a quadrilateral is a rectangle then the diagonals are of equal length and they bisect each other. **Converse statement:** If the diagonals of a quadrilateral are of equal length and bisect each other then the quadrilateral is a rectangle. (In this case the converse is true.) **Statement:** If $x = 2$ then $x^2 = 4$. **Converse statement:** If $x^2 = 4$ then $x = 2$. (In this case the converse is false.) **Statement:** If an animal is a kangaroo then it is a marsupial. **Converse statement:** If an animal is a marsupial then it is a kangaroo. (In this case the converse is false.) **Statement:** If a quadrilateral is cyclic then the opposite angles are supplementary. **Converse statement:** If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic. (In this case the converse is true.)

Contrapositive	The contrapositive of the statement 'If P then Q' is 'If not Q then not P'. The contrapositive of a true statement is also true. (not Q is the **negation** of the statement Q) Examples: **Statement:** A rectangle is a quadrilateral that has diagonals of equal length and the diagonals bisect each other. **Contrapositive:** If the diagonals of a quadrilateral are not of equal length or do not bisect each other then the quadrilateral is not a rectangle. **Statement:** If $x = 2$ then $x^2 = 4$. **Contrapositive:** If $x^2 \neq 4$ then $x \neq 2$. **Statement:** A kangaroo is a marsupial. **Contrapositive:** If an animal is not a marsupial then it is not a kangaroo. **Statement:** The opposite angles of a cyclic quadrilateral are supplementary **Contrapositive:** If the opposite angles of quadrilateral are not supplementary then the quadrilateral is not cyclic.
Counter example	A Counterexample is an example that demonstrates that a statement is not true. Examples: **Statement**: If $x^2 = 4$ then $x = 2$. **Counterexample**: $x = -2$ provides a counterexample. **Statement**: If the diagonals of a quadrilateral intersect at right angles then the quadrilateral is a rhombus. **Counterexample**: A kite with the diagonals not bisecting each other is not a rhombus. Such a kite provides a counterexample to the statement. The diagonals of a kite do intersect at right angles. **Statement**: Every convex quadrilateral is a cyclic quadrilateral. **Counterexample**: A parallelogram that is not a rectangle is convex, but not cyclic.
Equivalent statements	Statements P and Q are equivalent if $P \Rightarrow Q$ and $Q \Rightarrow P$. The symbol $\Leftrightarrow$ is used. It is also written as P if and only if Q or P iff Q. Examples: A quadrilateral is a rectangle if and only if the diagonals of the quadrilateral are of equal length and bisect each other. A quadrilateral is cyclic if and only if opposite angles are supplementary
Implication	If P then Q Symbol: $P \Rightarrow Q$ Examples: If a quadrilateral is a rectangle then the diagonals are of equal length and they bisect each other. If $x = 2$ then $x^2 = 4$ If an animal is a kangaroo then it is a marsupial. If a quadrilateral is cyclic then the opposite angles are supplementary.

Inverse	The inverse of the statement 'If P then Q' is 'If not P then not Q'. Symbolically the inverse of P $\Rightarrow$ Q is: $\neg$P$\Rightarrow$ $\neg$Q The inverse of a true statement need not be true. **Statement:** If an animal is a kangaroo then it is a marsupial. **Inverse statement:** If an animal is **not** a kangaroo then it is **not** a marsupial. (In this case the inverse is false.)
Negation	If P is a statement then the statement 'not P', denoted by $\neg$P is the negation of P. If P is the statement 'It is snowing.' then $\neg$P is the statement 'It is not snowing.'
Quantifiers For all (For each)	
Symbol $\forall$	For all real numbers $x, x^2 \geq 0$ can be written $\forall$ real numbers $x, x^2 \geq 0$. For all triangles the sum of the interior angles is 180° can be written, $\forall$ triangles the sum of the interior angles is 180°.
There exists **Symbol $\exists$**	There exists a real number that is not positive ($\exists$ a real number that is not positive.) There exists a prime number that is not odd. ($\exists$ a prime number that is not odd.) There exists a natural number that is less than 6 and greater than 3. There exists an isosceles triangle that is not equilateral. The quantifiers can be used together. For example: $\forall x \geq 0, \exists y \geq 0$ such that $y^2 = x$.
Geometry	
Alternate segment	The word 'alternate' means 'other'. The chord *AB* divides the circle into two segments and *AU* is tangent to the circle. Angle *APB* 'lies in' the segment on the other side of chord *AB* from angle *BAU.* We say that it is in the **alternate segment**. 
Cyclic quadrilateral	A **cyclic quadrilateral** is a quadrilateral whose vertices all lie on a circle. 

Lines and line segments associated with circles	Any line segment joining a point on the circle to the centre is called a **radius**. By the definition of a circle, any two radii have the same length called the radius of the circle. Notice that the word 'radius' is used to refer both to these intervals and to the common length of these intervals.  An interval joining two points on the circle is called a **chord**. A chord that passes through the centre is called a **diameter**. Since a diameter consists of two radii joined at their endpoints, every diameter has length equal to twice the radius. The word 'diameter' is used to refer both to these intervals and to their common length. A line that cuts a circle at two distinct points is called a **secant**. Thus a chord is the interval that the circle cuts off a secant, and a diameter is the interval cut off by a secant passing through the centre of a circle.
Important theorems	
Circle Theorems	**Result 1** Let *AB* be a chord of a circle with centre *O*. The following three lines coincide: The bisector of the angle $\angle AOB$ subtended at the centre by the chord. The line segment (interval) joining *O* and the midpoint of the chord *AB*. The perpendicular bisector of the chord *AB*. 
	Result 2 Equal chords of a circle subtend equal angles at the centre. In the diagram shown $\angle AOB = \angle POQ$. 

	Result 3 An angle in a semicircle is a right angle. Let *AOB* be a diameter of a circle with centre *O*, and let *P* be any other point on the circle. The angle $\angle APB$ subtended at *P* by the diameter *AB* is called an angle in a semicircle.  Converse: The circle whose diameter is the hypotenuse of a right-angled triangle passes through all three vertices of the triangle.
	Result 4 An angle at the circumference of a circle is half the angle subtended at the centre by the same arc. In the diagram shown $\angle AOB = 2\angle APB$  The arc *AB* subtends the angle $\angle AOB$ at the centre. The arc also subtends the angle $\angle APB$, called an angle at the circumference subtended by the arc *AB*.
	Result 5 Two angles at the circumference subtended by the same arc are equal. $\angle APB = \angle AQB$ In the diagram, the two angles $\angle APB$ and $\angle AQB$ are subtended by the same arc *AB*. 
	Result 6 The opposite angles of a cyclic quadrilateral are supplementary. Converse If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

	Result 7 Alternate segment theorem An angle between a chord and a tangent is equal to any angle in the alternate segment. In the diagram $\angle BAU = \angle APB$. 
UNIT 2	
Trigonometry	
Cosine and sine functions	Since each angle θ measured anticlockwise from the positive *x*-axis determines a point P on the unit circle, we will define the cosine of θ to be the x-coordinate of the point *P* the sine of θ to be the y-coordinate of the point *P* the tangent of θ is the gradient of the line segment *OP* 
Identities	**Angle sum and difference identities** $\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$ $\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$ **Products as sums and differences** $\cos A\cos B = \frac{1}{2}(\cos(A-B)+\cos(A+B))$ $\sin A\sin B = \frac{1}{2}(\cos(A-B)-\cos(A+B))$ $\sin A\cos B = \frac{1}{2}(\sin(A+B)+\sin(A-B))$ $\cos A\sin B = \frac{1}{2}(\sin(A+B)-\sin(A-B))$
Reciprocal trigonometric functions	$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$ $\operatorname{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$ $\cot A = \frac{1}{\tan A},\ \sin A \neq 0$

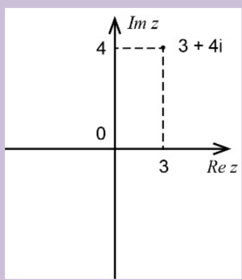
Trigonometric identities	**Double angle formula** $\sin 2A = 2\sin A\cos A$ $\cos 2A = \cos^2 A - \sin^2 A$ $= 2\cos^2 A - 1$ $= 1 - 2\sin^2 A$ $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$	**Pythagorean identities** $\cos^2 A + \sin^2 A = 1$ $1 + \tan^2 A = \sec^2 A$ $\cot^2 A + 1 = \text{cosec}^2 A$

Matrices

Addition of matrices (See Matrix)	If $\mathbf{A}$ and $\mathbf{B}$ are matrices with the same dimensions and the entries of $\mathbf{A}$ are a_{ij} and the entries of $\mathbf{B}$ are b_{ij} then the entries of $\boldsymbol{A} + \boldsymbol{B}$ are $a_{ij} + b_{ij}$ For example if $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 6 \end{bmatrix}$ then $\mathbf{A}+\mathbf{B} = \begin{bmatrix} 7 & 2 \\ 2 & 4 \\ 2 & 10 \end{bmatrix}$
Determinant	Determinant of a 2×2 matrix (See Matrix) If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the determinant of A denoted as $\det A = ad - bc$. If $\det\mathbf{A} \neq 0$, then; the matrix $\mathbf{A}$ has an inverse. the simultaneous linear equations $ax + by = e$ and $cx + dy = f$ have a unique solution. The linear transformation of the plane, defined by A maps the unit square $O(0,\ 0), B(0,1), C(1,\ 1), D(1,\ 0)$ to a parallelogram $OB'C'D'$ of area = $\lvert\det A\rvert$. The sign of the determinant determines the orientation of the image of a figure under the transformation defined by the matrix.
Dimension (or size) (See Matrix)	Two matrices are said to have the same **dimensions** (or **size**) if they have the same number of rows and columns. For example, the matrices $\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ and $\begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ have the same dimensions. They are both 2×3 matrices. An $m \times n$ matrix has m rows and n columns.
Entries (Elements) of a matrix	The symbol a_{ij} represents the (i, j) entry which occurs in the i^{th} row and the j^{th} column. For example a general 3×2 matrix is: $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and a_{32} is the entry in the third row and the second column.
Leading diagonal	The leading diagonal of a square matrix is the diagonal which runs from the top left corner to the bottom right corner.

Linear transformations defined by a 2 × 2 matrix	The matrix multiplication $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$ defines a transformation $T(x,y) = (ax+by, cx+dy)$
Linear transformations in 2 dimensions	A linear transformation in the plane is a mapping of the form $T(x,y) = (ax+by, cx+dy)$ A transformationT is linear if and only if $T(\alpha(x_1,y_1)+\beta(x_2,y_2)) = \alpha T(x_1,y_1)+\beta T(x_2,y_2)$ Linear transformations include: • rotations around the origin • reflections in lines through the origin • dilations Translations are not linear transformations.
Matrix (matrices)	A **matrix** is a rectangular array of elements or entries displayed in rows and columns. For example, $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ are both matrices. Matrix **A** is said to be a 3 × 2 matrix (three rows and two columns) while **B** is said to be a 2 × 3 matrix (two rows and three columns).
Square matrix	A **square matrix** has the same number of rows and columns.
Column matrix	A **column matrix** (or vector) has only one column.
Row matrix	A **row matrix** (or vector) has only one row.
Matrix algebra of 2 × 2 matrices	If **A, B** and **C** are 2 × 2 matrices, **I** the 2 × 2 (multiplicative) identity matrix and **O** the 2 × 2 zero matrix then: **A** + **B** = **B** + **A** (commutative law for addition) (**A** + **B**) + **C** = **A** + (**B** + **C**) (associative law for addition) **A** + **O** = **A** (additive identity) **A** + (−**A**) = **O** (additive inverse) (**AB**)**C** = **A**(**BC**) (associative law for multiplication) **AI** = **A** = **IA** (multiplicative identity) **A**(**B** + **C**) = **AB** + **AC** (left distributive law) (**B** + **C**)**A**= **BA** + **CA** (right distributive law)

Matrix multiplication	**Matrix multiplication** is the process of multiplying a matrix by another matrix. The product $\mathbf{AB}$ of two matrices $\mathbf{A}$ and $\mathbf{B}$ with **dimensions** $m \times n$ and $p \times q$ is defined if $n = p$. If it is defined, the product $\mathbf{AB}$ is an $m \times q$ matrix and it is computed as shown in the following example. $$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}\begin{bmatrix} 6 & 10 \\ 11 & 3 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 94 & 34 \\ 151 & 63 \end{bmatrix}$$ The entries are computed as shown $1 \times 6 + 8 \times 11 + 0 \times 12 = 94$ $1 \times 10 + 8 \times 3 + 0 \times 4 = 34$ $2 \times 6 + 5 \times 11 + 7 \times 12 = 151$ $2 \times 10 + 5 \times 3 + 7 \times 4 = 63$ The entry in row i and column j of the product ***AB*** is computed by 'multiplying' row i of ***A*** by column j of **B** as shown. If $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$ then $\mathbf{AB} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$
(Multiplicative) identity matrix	A **(multiplicative) identity matrix** is a square matrix in which all the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter ***I***. For example, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ are both identity matrices. There is an identity matrix for each order of square matrix. When clarity is needed, the order is written with a subscript: I_n
Multiplicative inverse of a square matrix	The inverse of a square matrix $\mathbf{A}$ is written as $\mathbf{A}^{-1}$ and has the property that $$\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$ Not all square matrices have an inverse. A matrix that has an inverse is said to be **invertible**.
Multiplicative inverse of a 2 × 2 matrix	The **inverse** of the matrix $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\mathbf{A}^{-1} = \dfrac{1}{\text{Det}\mathbf{A}}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, when det $A \neq 0$.
Scalar multiplication (matrices)	**Scalar multiplication (matrices)** is the process of multiplying a matrix by a scalar (number). For example, forming the product: $10\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 0 & 30 \\ 10 & 40 \end{bmatrix}$ is an example of the process of scalar multiplication. In general for the matrix ***A*** with entries a_{ij} the entries of $k\boldsymbol{A}$ are ka_{ij}.

Singular matrix	A matrix is singular if det $\boldsymbol{A} = 0$. A singular matrix does not have a multiplicative inverse.
Zero matrix	A zero matrix is a matrix if all of its entries are zero. For example: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ are zero matrices. There is a zero matrix for each **size** of matrix. When clarity is needed, we write $\mathbf{O}_{m \times n}$ for the $m \times n$ zero matrix.
Real and complex numbers	
Complex arithmetic	If $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$ $\Rightarrow z_1 + z_2 = (x_1 + x_2) + (y_1 + y_2)i$ $\Rightarrow z_1 - z_2 = (x_1 - x_2) + (y_1 - y_2)i$ $\Rightarrow z_1 \times z_2 = (x_1 x_2 - y_1 y_2) + (x_1 y_2 + x_2 y_1)i$ $z_1 \times (0 + 0i) = 0$ $\quad Note: 0 + 0i$ is usually written as 0 $z_1 \times (1 + 0i) = z_1$ $\quad Note: 1 + 0i$ is usually written as 1
Complex conjugate	For any complex number $z = x + iy$, its conjugate is $\overline{z} = x - iy$, the following properties hold $\overline{(z_1 z_2)} = \overline{z}_1 \times \overline{z}_2$ $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z}_1}{\overline{z}_2}$ $z \times \overline{z} = \|z^2\|$ $z + \overline{z}$ is real.
Complex plane (Argand plane)	The **complex plane** is a geometric representation of the complex numbers established by the **real axis** and the orthogonal **imaginary axis**. The complex plane is sometimes called the Argand plane. 
Imaginary part of a complex number	A complex number z may be written as $x + yi$, where x and y are real, and then y is the imaginary part of z. It is denoted by Im (z).
Integers	The integers are the numbers $\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$.
Modulus (Absolute value) of a complex number	If z is a complex number and $z = x + iy$ then the modulus of z is the distance of z from the origin in the Argand plane. The modulus of z denoted by $\|z\| = \sqrt{x^2 + y^2}$.
Prime numbers	A prime number is a positive integer greater than 1 that has no positive integer factor other 1 and itself. The first few prime numbers are $2, 3, 5, 7, 11, 13, 17, 19, 23, \cdots$.

Principle of mathematical induction	Let there be associated with each positive integer n, a proposition $P(n)$. $P(1)$ is true, and for all k, $P(k)$ is true implies $P(k+1)$ is true, then $P(n)$ is true for all positive integers n.
Rational numbers	A real number is **rational** if it can be expressed as a quotient of two integers. Otherwise it is called irrational. Irrational numbers can be approximated as closely as desired by rational numbers, and most electronic calculators use a rational approximation when performing calculations involving an irrational number.
Real numbers	The numbers generally used in mathematics, in scientific work and in everyday life are the **real numbers**. They can be pictured as points on a number line, with the integers evenly spaced along the line, and a real number a is to the right of a real number b if $a > b$ A real number is either rational or irrational. The set of real numbers consists of the set of all rational and irrational numbers. Every real number has a decimal expansion. Rational numbers are the ones whose decimal expansions are either terminating or eventually recurring.
Real part of a complex number	A complex number z may be written as $x + yi$ where x and y are real, and then x is the real part of z. It is denoted by $\mathrm{Re}(z)$.
Whole numbers	A **whole number** is a non–negative integer, that is, one of the numbers $0,1,2,3...$,